Worksheet 1:

Sailing with light

Introduction:

Photons have momentum that depends on the light's energy. Since $E_{Ph} = m_{Ph} \cdot c^2$ and $p = m \cdot c$, it follows that $p = \frac{E_{Ph}}{c}$. This momentum can be transferred to bodies hit by light.

The idea to use **radiation pressure from sunlight** or gigantic **laser systems** to propel space probes has been around for a while. If you stretch a very thin, highly reflective foil across huge sails, photons will bounce off the foil and transfer their momentum to the sails. This worksheet presents a concrete example of an application of this concept.

The momentum of photons can also be used at microscopically small scales to move small objects. Today, **optical tweezers**, whose invention was awarded the Nobel Prize in Physics in 2018, play an important role in medical and biological research. The second worksheet explains how these **light-based grippers** work, and the third worksheet discusses some of their typical applications.

Sailing to Proxima Centauri b with light

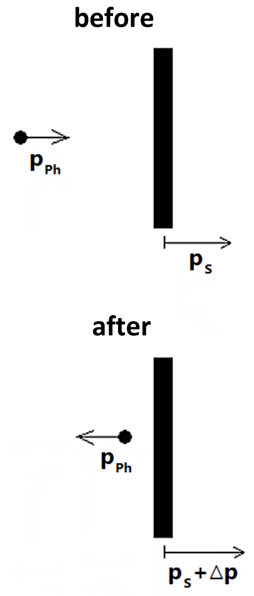
In 2016, an Earth-like exoplanet was discovered orbiting the habitable zone of the nearby star Proxima Centauri b. After this discovery, an extremely bold and visionary idea was published explaining how this planet, 4.2 light years away, could be reached by a space probe within a travel time of 20 years. To propel the space probe, an extremely lightweight but stable **light sail** was suggested. Even if the whole thought experiment might seem somewhat utopian, taking a closer look at the physics behind the project is quite interesting.

Exercises:

1. Find out more about the visionary project called "Breakthrough Starshot", for example at the following link: https://blogs.scientificamerican.com/life-unbounded/can-starshot-work/

2. Below, we will calculate how much **momentum** the photons can transfer to the light sails.

   Suppose that the spacecraft is travelling away from us at constant speed with momentum p_S. The sails are hit by photons with total momentum p_{Ph}. Since the mass of the spacecraft is much, much higher than the mass of the photons, the photons are reflected like from a wall – so they keep their momentum but change direction. This elastic collision transfers a momentum Δp to the sail.



To calculate the increase in the momentum, we can use the following equation:

$$p_S + p_{Ph} = (p_S + \Delta p) - p_{Ph}$$

a) Explain this equation.
Hint: Recall the law of momentum conservation.

b) Deduce that the momentum transfer from the photons to the sails is $\Delta p = 2 \cdot p_{Ph}$.

c) Light with energy E_{Light} has momentum $p_{Ph} = \frac{E_{Light}}{c}$.
Using $\Delta p = m \cdot \Delta v$, show that the speed of the spacecraft of mass m_S increases by $\Delta v = \frac{2 \cdot E_{Light}}{m_S \cdot c}$ when hit by the light.

d) The "Starshot project" proposes to use a massive array of laser cannons to generate enormous light pressure on the sails of the space probe. The proposed light output of 100 gigawatts would radiate an energy per second of $E = 100 \cdot 10^9$ J.[1] Calculate the speed increase per second (acceleration) if we assume (rather unrealistically) that the total mass of the spacecraft is 1.33 grams, that all photons hit the sails, and that all photons are fully reflected.

e) Calculate the speed increase after 120 seconds of beam exposure. What percentage of the speed of light does the spacecraft reach?

<u>**Note:**</u> If your calculations are correct, you should find that the spaceship travels at 20 % of the speed of light. This is the value repeatedly cited by publications on the Starshot project. However, it is only possible if the total system mass is 1.33 grams, which seems somewhat unrealistic – not to mention the incredibly high laser power required. But purely in terms of physics, the idea of using the momentum of photons for an interstellar drive is completely plausible and rather fascinating.

[1] This is roughly the amount of electrical energy consumed by five single-family homes in a year.

