The Starting Situation for the Standardisation of the SI system

In 1798, the French Revolution demanded standardised units of measurement alongside the use of the decimal system. This led both to the introduction of the *mètre des Archives*, which was defined as one ten-millionth of the shortest distance from the North Pole to the equator passing through Paris, and also to the creation of the *International Prototype Kilogram*, which was intended to correspond to the mass of one litre or cubic decimetre of water at a temperature of 4° C.

In 1875 these two artefacts formed the basis of the international agreement at the Metre Convention, initially signed by 17 founder states – including France, Russia and the German Empire – in order to create standardised units for the future. Since 2015, the Metre Convention has 60 member states, as well as another 40 states and global organisations. However, the use of metric units has not yet become fully established around the globe. For example, in the USA lengths, masses and temperatures are still measured in miles, ounces and degrees Fahrenheit.

The last two decades, with their scientific and technical developments, demonstrated that inadequacies in the definition of units – of the kilogram in particular – were having increasingly noticeable negative effects. Deviations in the prototype kilogram led to intolerable changes in all other measurements that depended on the kilogram.

The simplified SI system introduced in 2019 uses unchangeable – according to current knowledge – natural constants for science and technology on the basis of measurements and calculations – independent of arbitrary and imprecise dimensions.

Note:

The video "What are International Units of Measurement?" gives a general overview of the SI system and its historical development:

https://www.mediatheque.lindau-nobel.org/videos/38525/si-units-i-en

1. Natural Constants for Time and Length

See also: https://www.mediatheque.lindau-nobel.org/videos/38527/si-units-iii-en

a) Definition of time

Since 1967, the measurement of the **second** has been defined as the transition between the two *hyperfine levels of the ground state of atoms of the Caesium isotope ^{133}Cs*. This means one second is equivalent to a multiple of 9,192,631,770 of the periodic time of the radiation thus emitted.

The **defined natural constant** here is the frequency $\Delta \nu = 9192631770\ s^{-1}$.

This results in the **definition of the base unit**: $1s = \frac{9192631770}{\Delta \nu(^{133}Cs)}$

b) Definition of length

In 1983, one **metre** was defined as the *speed of light* at exactly $c = 299792458\ m \cdot s^{-1}$. Consequently, one metre is the length of the path travelled by light in a vacuum during a time interval of 1/299,792,458 of a second.

Show with a calculation that the following figures result for the **definition of the base unit**:

$$1\,m\ =\ 30.663318 \ldots \frac{c}{\Delta \nu(^{133}Cs)}$$

c) Demonstrate why the definitions of second and metre are connected by the natural constants c.

2. The Redefinition of the Kilogram

a) Problems with and because of the kilogram

Until 2019, the "International Prototype of the Kilogram", stored in a safe at the International Bureau of Weights and Measures (BIPM) on the outskirts of Paris (**Ill. 1**), as well as its official and numerous national copies, were the basis for all masses. But when the masses were compared, it was found that almost all the copies were heavier than the prototype – until now nobody can prove exactly why this was the case. However, it was clear that the prototype kilogram had to be replaced – after all, other units depend on the prototype kilogram, such as the *ampere* and *mole*, with the result that these units also had problems due to the kilogram.

Illustration 1: Photo: Japs 88 – wikipedia.org

b) Redefinition of the kilogram by defining the Planck constant h

See also: https://www.mediatheque.lindau-nobel.org/videos/38526/si-units-ii-en

To redefine the **kilogram**, a natural constant was selected whose unit includes the kg and whose measured value can be determined with extremely high precision – the decision was made to adopt the *Planck constant h*:

$$h = 6.62607015 \cdot 10^{-34} Js = 6.62607015 \cdot 10^{-34} \frac{kg \cdot m^2}{s}$$

Two experiments were conceived and implemented to realise a future stable kilogram:

- Using what is known as the *Kibble balance experiment*, the effect of gravity on a test object is compensated by an electro-magnetic force, which allows a figure for h to be deduced.
- In parallel, a perfectly polished crystal ball made of high-purity silicon was built from the mass of 21.442... quadrillion atoms, whose number can be counted with very high precision due to the crystal structure. This experiment, which is called the *Avogadro experiment,* results in the “Avogadro constant N_A”, from which the Planck constant can be calculated.
- This means that the results from the *Kibble balance experiment* and the silicon ball can be compared – as soon as the results from both experiments were consistent, the path to the new kilogram was clear.

Task: Research in textbooks and on the internet what is meant by the “Avogadro constant” and “Planck constant” and find out about the two experiments to redefine the kilogram.

Tip: The following YouTube video will provide initial explanations:

https://www.youtube.com/watch?v=m-fFRLWBzm8

c) The figure for the Planck constant is

$$Js = 6.62607015 \cdot 10^{-34} \frac{kg \cdot m^2}{s}.$$

Solve this equation for the unit kg and explain your result with reference to the Planck constant – also include **Ill. 2**.

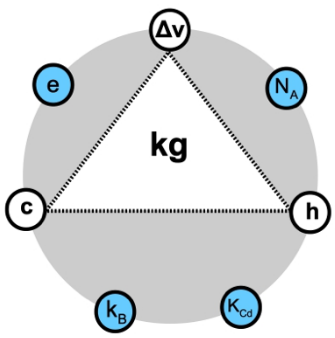

Illustration 2: Wolfgang Vogg

For an explanation of the definition of the unit ampere as well as the following definitions of kelvin, mole and candela **see also:**

https://www.mediatheque.lindau-nobel.org/videos/38528/si-units-iv-en

1. A New Basis for the Ampere

The **ampere** was chosen as the unit to measure the strength of electrical current – it is the only electrical base unit in the International System of Units. The version that has applied since 1948 is depicted in **Ill. 1**:

The base unit of 1 ampere is defined as the temporary constant current which, if maintained between two straight parallel conductors of infinite length, and placed 1 metre apart, generates a force equal to $2 \cdot 10^{-7}$ N per metre of length.

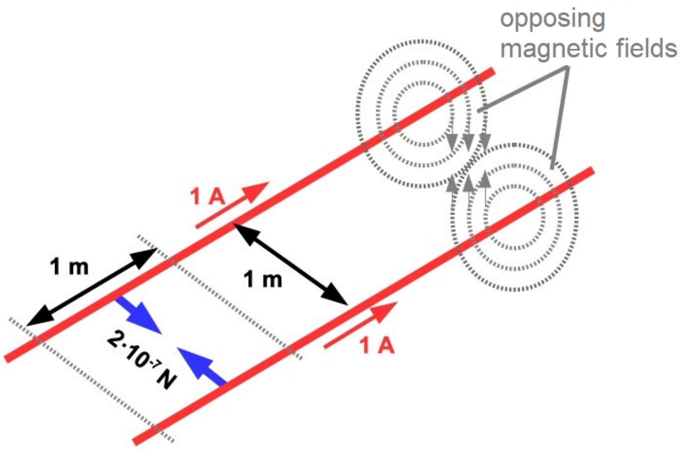

Illustration 1: Wolfgang Vogg

This arbitrarily chosen and unrealistic measurement specification could only be carried out approximately. In addition, it has the major disadvantage that it connects the ampere with the kilogram through force, with the problems that have already been mentioned.

For this reason, a different approach was taken decades ago:

In 1962, the British theoretical physicist and 1973 Nobel Laureate **Brian D. Josephson** had already predicted an **effect in superconductors** which made it possible to measure electrical currents with a high degree of accuracy. In addition, in 1980, the German physicist and 1985 Nobel Laureate **Klaus von Klitzing** discovered the **quantum Hall effect**, which enabled an exceedingly precise quantisation of electrical resistance.

a) Find out about the discoveries made by the two physicists and explain why their findings were not fully sufficient for a redefinition of the ampere.

Today, after the reform of the SI system, the ampere is also based on a natural constant – the elementary charge of the electron. In complex measurements, it has been possible to define the ampere through the electrical current of well above 1 trillion elementary charges per second.

The following applies to the elementary charge: $e = 1.602176634^{-19} A \cdot s$

b) Solve this equation for the unit A and calculate its value with the help of the corresponding natural constant.

2. Definition of the Remaining Base Units of Kelvin (K), Mole (Mol), Candela (cd)

a) The kelvin (K) as a unit of temperature

The **kelvin** is the SI unit of thermodynamic temperature. The *kelvin scale* is no different to the everyday *Celsius temperature scale* with a shifted absolute zero, meaning that -273.15 °C equals 0 K. Consequently, there are no negative temperatures when using the unit kelvin and one kelvin step equals one Celsius step.

The kelvin is defined as the natural constant k_B, which is called the *Boltzmann constant*:

$$k_B = 1.3806488 \cdot 10^{-23} \frac{J}{K}$$

α) Research how the "Boltzmann constant" was determined in your textbooks and on the internet.

β) Explain the significance of $k_B = 1.3806488 \cdot 10^{-23} \frac{J}{K}$, if the temperature is changed by1 K.

γ) Calculate the applicable relationship for 1 K. Also include **Ill.2**.

$$1K = 2.2666653 \ldots \frac{\Delta v(^{133}\,Cs)h}{k_B}$$

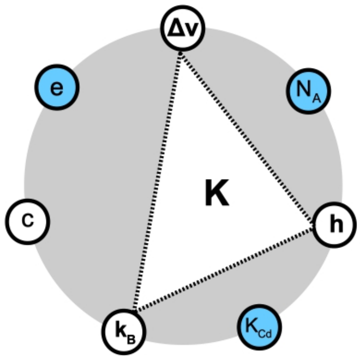

Illustration 2: Wolfgang Vogg

b) The mole (mol) as a unit of the amount of substance

By setting the kilogram on the Planck constant h, it was possible to determine the Avogadro constant N_A.

In turn, it became possible to relate the mole to a natural constant, so that the following applies today:

The **mole** is the unit of an amount of substance containing $6.02214076 \cdot 10^{23}$ particles of the same kind, which may relate to atoms, molecules, ions, electrons or other particles. Thus, the following applies to 1 mole:

1 mole = $\frac{6.02214076 \cdot 10^{23}}{N_A}$

c) The candela (cd) as the unit of luminous intensity

Luminous intensity – defined by the unit **candela** – hardly comes up in lessons in upper secondary school, but is extremely important as a unit in natural science and the resulting technical applications. Therefore, its definition and classification in the new SI system will only be introduced here – primarily to stimulate interest:

Luminous intensity was once derived from the flame of a candle with a certain wick height. By using such standardised candles, it was possible to determine how brightly a source of light was shining.

Since 1979, the unit candela (cd) – Latin for candle – has been defined by the luminous intensity of a green source of light with a frequency λ = 555 nanometres that at a certain output (1/683 watts) emits electro-magnetic radiation in a certain solid angle.

One candela corresponds approximately to the luminous intensity of one household candle. Using a conversion factor, the *photometric radiation equivalent* K_{cd}, the measurement, which is actually adjusted to the light sensitivity of the human eye, is connected to electro-magnetic radiation physics. The fact that the candela has survived as a unit is a concession to the lighting industry. Therefore, the definition will not be changed in the future.

There is the following correlation for 1 cd: $1cd = \left(\frac{K_{cd}}{683}\right) kg \cdot m^2 \cdot s^{-3} \cdot sr^{-1}$ *

* *Steradian (sr)* is a unit of measurement for the solid angle – projected onto a sphere with a radius of 1 m, a steradian encloses an area of 1 m^2 on the surface of the sphere.

Additional natural constants are included as follows:

$$1cd = \frac{1}{(6.62607015 \cdot 10^{-34}) \cdot (9192631770)^2} \; [\Delta v(^{133}\,Cs)]^2 \, h \, K_{cd}$$

$$1cd = 2.614830 \ldots \cdot 10^{10} \; [\Delta v(^{133}\,Cs)]^2 \, h \, K_{cd}$$

Thus, one candela is the luminous intensity of a source of radiation in a particular direction that emits a frequency of $540 \cdot 10^{12}$ Hz and has a radiant intensity in this direction of 1/683 W sr^{-1}.

3. Overview of All Base Units and the Associated Natural Constants

Ill. 3 shows to which natural constant the respective base unit refers and the relationship between the individual units of measurement.

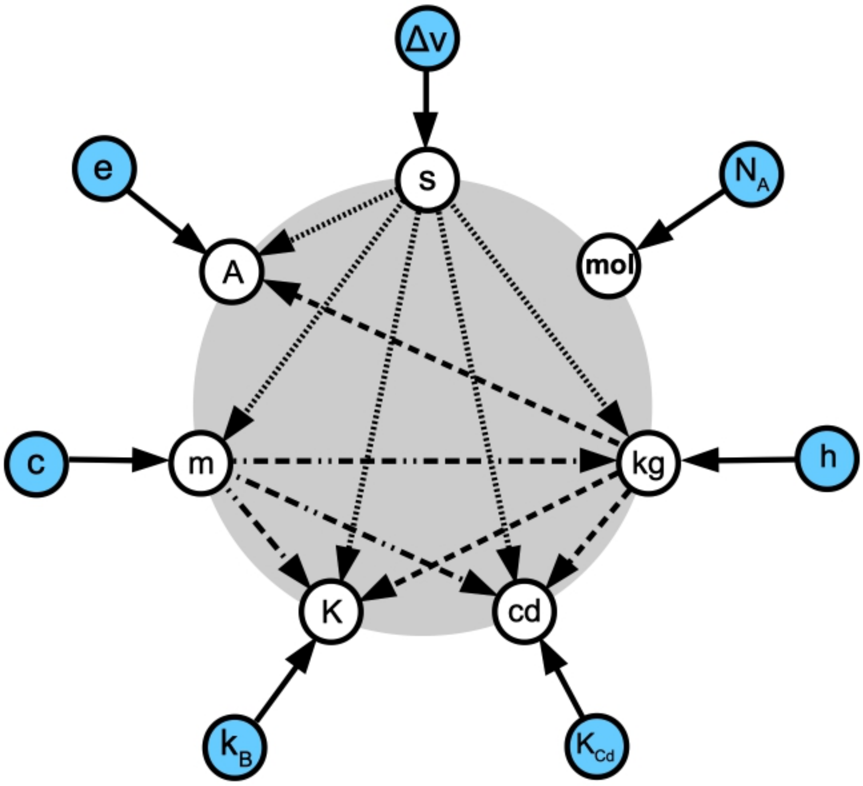

Illustration 3: Wolfgang Vogg

Disentangle the linkage using the differently labelled arrows and discover the connections between the individual base units and their associated natural constants.

