The Physical Background of Black Holes

A. What are black holes? How are they formed?

In the course of billions of years, stars, galaxies, and entire clusters of galaxies – more than one hundred billion – have formed in the universe. These, in turn, consist of millions and millions of stars and radiate in a cosmos that has exceeded a size of over 10^{23} km.

1. The development into different star types and their further course in their life cycle are shown in **Fig. 1 a and b.**

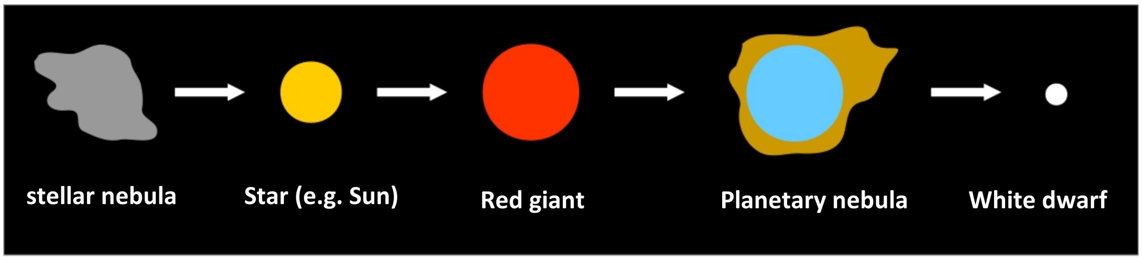


Fig. 1 a: Life cycle of a smaller star (e.g. the sun). Source: W. Vogg

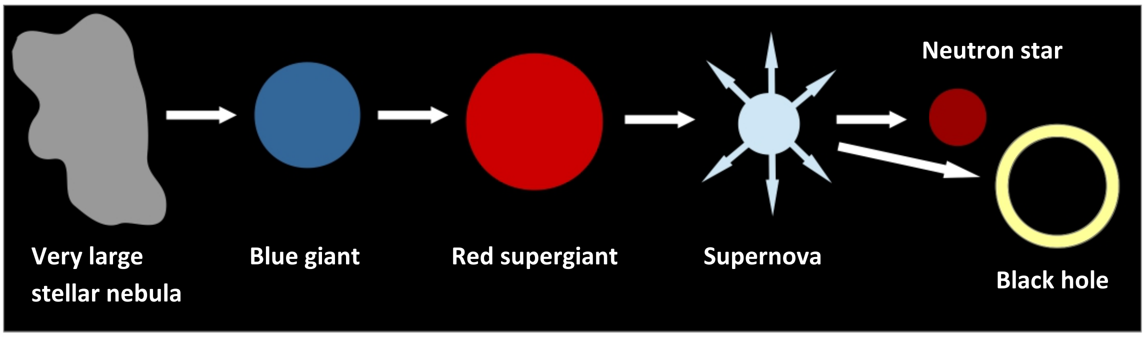


Fig. 1 b: Life cycle of a giant star. Source: W. Vogg

a) Describe and explain the respective life cycle of a star
   α) from a small stellar nebula to a white dwarf (Fig. 1 a).
   β) from a much larger stellar nebula to a supernova (Fig. 1 b).

b) Explain why a supernova can turn into a neutron star and, under certain conditions, into a black hole.

Note: Research this in suitable text books and/or on the internet!

B. How can you picture the structure of black holes?

The term **"Black hole"** is used to describe gigantic masses squeezed together in the universe. These distort the *space-time structure* around them through their gravitational field in such a way that neither matter nor light can escape from this area to the outside. These objects are thus also not visible.

The *General Theory of Relativity* published in 1915 by **Albert Einstein** (1879–1955) states that massive objects bend the **space-time** surrounding them because of their gravitational field. Einstein's prediction was clearly proven during a total solar eclipse in 1919, when astronomers measured the displacement of the light rays coming from the stars while the moon covered the solar disk.

A black hole comprises so much concentrated mass that it causes said space-time to collapse completely. It was *Roger Penrose* who, after many years of theoretical research, published a paper in 1965 that ultimately won him the Nobel Prize in Physics in 2020. With the help of topological methods and extremely complicated mathematics, he was able to show that when stars implode at the end of their lifetime, a point at which nothing opposes gravity can be passed, and a **singularity** is created.

In doing so, he developed the concept of a "trapped surface", which can be imagined as a two-dimensional, closed structure – similar to a ring – forcing all radiation inwards towards the centre. Inside the ring is the point-like singularity in which space and time can no longer be defined.

With this concept, Roger Penrose explains why light and matter only pass through the **event horizon (also called the Schwarzschild radius)** of a black hole in only one direction (i.e. inwards). Space and time exchange their roles inside this surface, whereby the inward movement becomes a forward movement in time. Precisely this consideration makes a return from the black hole impossible because that would be a time travel into the past.

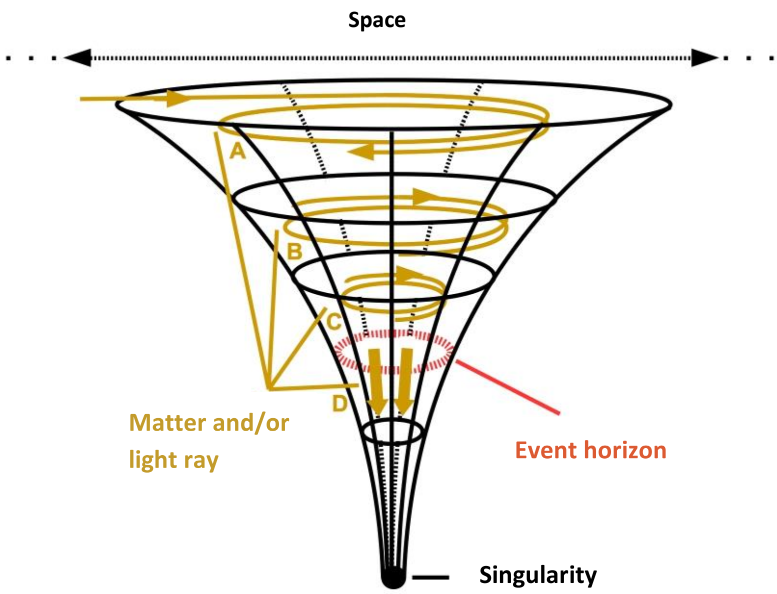


Fig. 2: Structure and functioning of a black hole. Source: W. Vogg

2. **Fig. 2** shows how one can picture the build-up to a black hole – starting from the already curved gravitational field in space via the event horizon to the singularity.
   a) First explain what is meant by the terms space-time, event horizon, Schwarzschild radius, and singularity.

b) Now use Fig. 2 (A–D) to describe how the capture of matter and light proceeds under the influence of an increasingly strong gravitational force towards the singularity of the black hole.

3. Despite the enormous importance of the General Theory of Relatively for the description of black holes, the Schwarzschild radius of a black hole can be calculated using the classical physics of a spherical body. Such a body with a mass M and a radius R has a radially symmetrical gravitational field. If you want to leave the gravitational field of such a body, you need the following escape velocity (2nd cosmic velocity):

$$V_{FI} = \sqrt{\frac{2 \cdot G \cdot M}{R}}$$

a) Derive the equation for the escape velocity.

b) Calculate the formula for the Schwarzschild radius RS by considering the speed of light c as the greatest possible escape velocity.

c) Calculate the Schwarzschild radius RS for the Sun and the Earth assuming that both celestial bodies could become black holes.

Use the following values for your calculations:

Speed of light:	$c = 2.998 \cdot 10^{8}\,\frac{m}{s}$
Mass of the Sun:	$m_{So} = 1.989 \cdot 10^{30}\,kg$
Mass of the Earth	$m_{E} = 5.972 \cdot 10^{24}\,kg$
Gravitational constant:	$G = 6.674 \cdot 10^{-11}\,\frac{m^3}{kg \cdot s^2}$

C. The detection of black holes

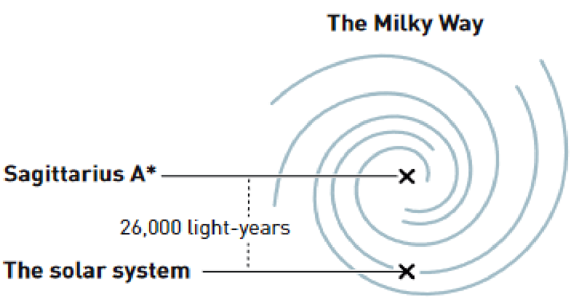
In the 1990s, astrophysicists **Reinhard Genzel** and **Andrea Ghez** independently decided to focus their research on our own galaxy, the Milky Way, with the aim of detecting a black hole at the centre.

Together with their research groups and through years of observations and recordings, both were able to consistently identify the orbits of several stars that rotate elliptically around a centre at very high speed – an extremely difficult undertaking because the centre is hidden in visible light by gas and dust clouds and can be observed only with large telescopes in the infra red range.

Fig. 3 shows the Milky Way in plan view – it appears in the sky as a flat disk with a huge diametre of about 100,000 light years.

Its various spiral arms consist of gas and dust as well as several hundred billion stars – one of which is our sun with the planets of the solar system.

Both Genzel and Ghez succeeded in detecting an extremely bright and compact object near the constellations *Sagittarius* and *Scorpio*. This was named **Sagittarius A* (SgrA*)**. To do this, both teams had tracked several stars tracing their orbits near SgrA* over two decades. A star with the designation S2 was believed to be particularly interesting **(Fig. 4**).



The Milky Way, our galaxy, seen from above. It is shaped like a flat disc about 100,000 light-years across. Its spiral arms are made of gas and dust and a few hundred billion stars. One of these stars is our Sun.

©Johan Jarnestad/The Royal Swedish Academy of Sciences

Fig. 3: The Milky Way in plan view
Source: https://www.nobelprize.org/uploads/2020/10/fig3-phy-en-milky-way.pdf

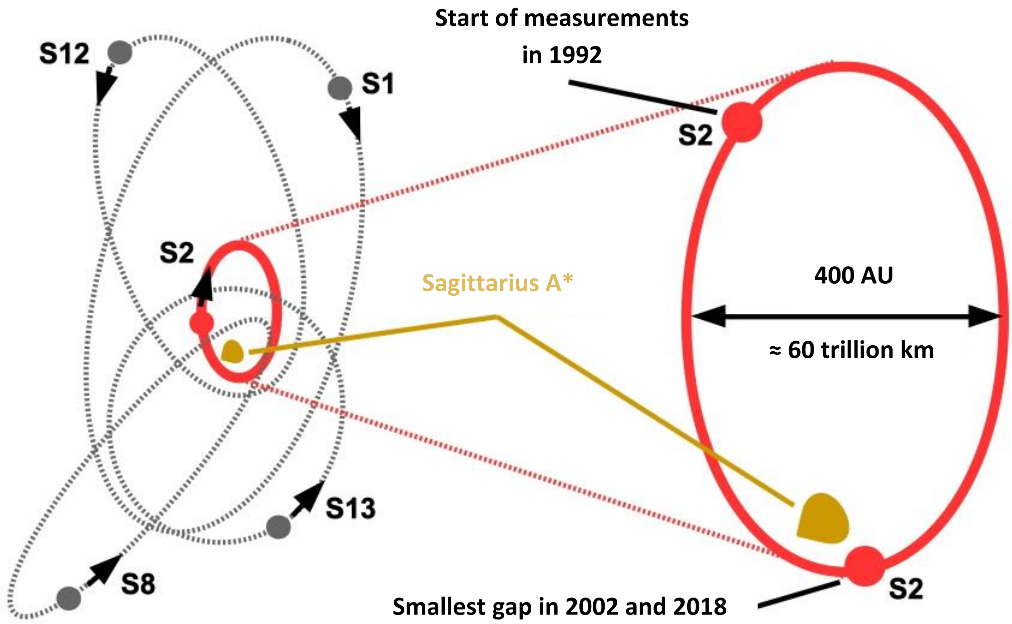


Fig. 4: Sagittarius A and the star S2. Source: W. Vogg*

It takes 16 years to orbit the galactic centre and approaches it up to 125 astronomical units (125 AU ≈ 18.75 trillion km). From the velocity measurements of Genzel and Ghez, it became clear that there must be a concentrated object within the orbit of S2 and that this must have a *mass of four million solar masses*.

4. Using Kepler's third law, which compares the orbital periods of different planets around the same central body, we can calculate the mass of a central body if we know the radius r (in the case of circular motion) or the major semi-axis a (in the case of elliptical motion). The latter applies to the motion of the star S2 around the central body SgrA*.

a) The centripetal force necessary for a circular and/or elliptical orbit, which is directed towards the centre of the circle of curvature, corresponds to the gravitational force of the central body – in our case, the black hole SrgA*: FZ = FGr

   Task: Derive from this, and by using the formula for the orbital velocity on an ellipse $V_{B,E} = \frac{2\pi \cdot a}{T}$, Kepler's third law: $\frac{a^3}{T^2} = \frac{G \cdot M}{4 \cdot \pi^2}$

b) Now calculate the mass of the black hole SgrA* from Kepler's third law using the value $a = 1.526 \cdot 10^{14}\ \mathrm{m}$ for the major semi-axis and 16.05 years for the orbital period.

c) How many solar masses does SgrA* contain?

d) Calculate the Schwarzschild radius of SgrA* as well as its diametre and compare it with the diametre of the Sun.

D. The importance of gravitational waves

As early as 1916, Albert Einstein postulated gravitational waves in the form of distortions of space-time as a direct consequence of his general theory of relativity. After a finite time, these can affect distant locations because of their propagation speed.

In September 2015, the researchers of the **LIGO project** (see box below) provided the first direct evidence. The American physicists **Rainer Weiss** (*1932), **Kip S. Thorne** (*1940), and **Barry C. Barish** (*1936) were jointly awarded the Nobel Prize in Physics for this achievement in 2017.

LIGO is the largest gravitational wave observatory and one of the most sophisticated physics experiments in the world. Consisting of two *laser interferometers* located thousands of kilometres apart in the US, *in Livingston, Louisiana*, and *Hanford, Washington,* LIGO uses the physical properties of light and space itself in order to detect gravitational waves.

A first network of interferometers was ready in the early 2000s: TAMA300 in Japan, GEO600 in Germany, LIGO in the US, and Virgo in Italy. Joint observations between 2002 and 2011 did not lead to the detection of any gravitational waves.

After fundamental improvements, the LIGO detectors began operating in 2015 as **Advanced LIGO** – the first in a far more sensitive global network of advanced detectors.

In August 2017, astronomers were able to receive both electromagnetic radiation and gravitational waves from a particular event. The event came from the galaxy NGC 4993, 130 million light years away – where two neutron stars had collided and merged. Finally, in October 2020, the American **Advanced LIGO** and the **Italian–French Advanced Virgo collaborations** published a gravitational wave catalogue, which now includes 50 gravitational wave signals, all of which can be traced back to merging black holes.

5. **Fig. 5** shows a schematic experimental set-up for measuring gravitational waves from the American LIGO project.
   Note: Do some research on the internet to answer the following questions.
   a) Using the sequence of numbers 1 to 4 in Fig. 5, describe the experimental set-up for detecting gravitational waves.

   b) Briefly explain the normal operation of the system as well as the measurement results by gravitational waves (Fig. 5 right).

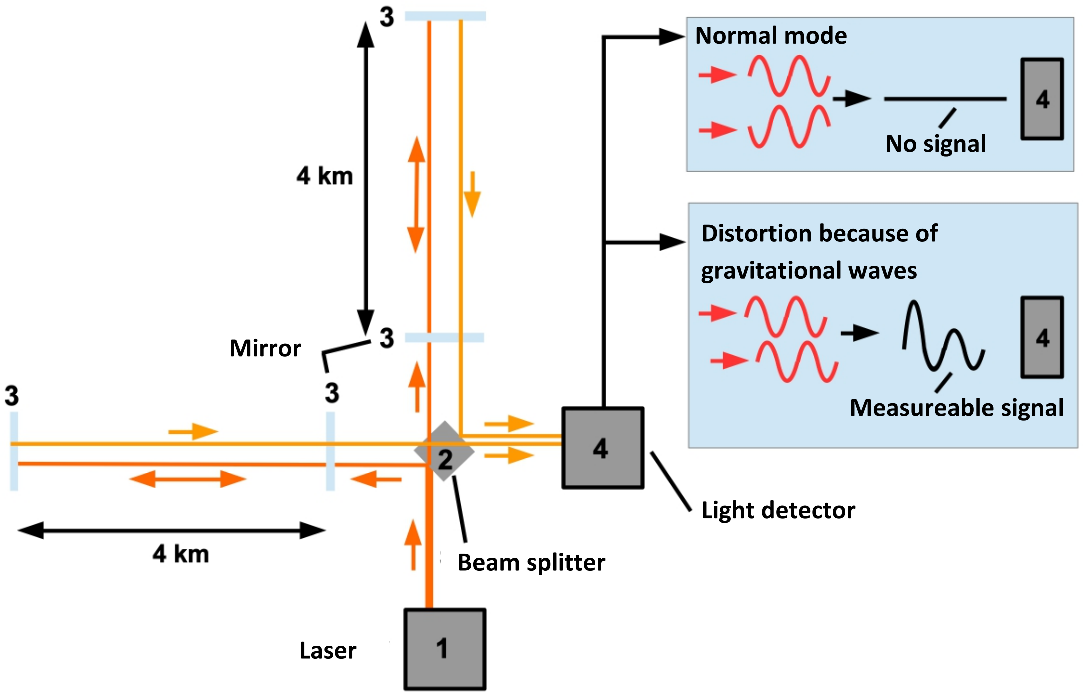


Fig. 5: Measuring arrangement for the detection of gravitational waves. Source: W. Vogg

E. Prospects

Astronomers estimate that there are probably several hundred million black holes in the Milky Way alone – including *supermassive black holes* such as Sagittarius A* weighing 10,000 to several billion solar masses, *intermediate black holes* of about 1,000 solar masses, *stellar black holes* of about 10 solar masses, and *primordial black holes* (comparable to the mass of the Earth's moon).

But only a few dozen have been discovered to date – not least because with the technical methods currently available black holes can be discovered only if there is a lot of matter in the catchment zone of the black holes and only if this is heated to extremely high temperatures by the attraction and subsequently emits measurable radiation. It is suspected that an extremely large number of silent black holes exist in the vicinity of which there is no matter that can emit radiation.

In addition, the question naturally arises as to whether all dark celestial objects are actually classical black holes. You may well be sceptical as to whether all the theories proposed by astrophysicists can become reality – rather, it will be the same in the future as in the past:

New theories have to prove themselves and be examined with scientific methods down to the smallest detail. If they are conclusive and completely convincing, that means that science has taken another step forward.

Black holes with their unimaginable effects on the interconnection of space and time will remain mysterious and literally inscrutable for a long time to come. They will also be an enormous incentive for creative natural scientists.